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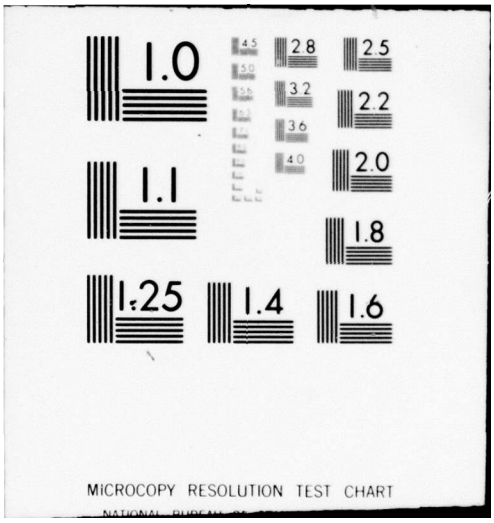
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CLASSIFICATION BY MARKOVIAN STRINGS OF PINGS

INTRODUCTION

In analyzing the performance of a decision scheme for sonar echo classification, C. S. Stradling and G. P. Schumacher (Reference 1) experimentally found the probability of correct identification (PCI) for a sequence of pings in a case in which the single ping detection and false-alarm probabilities are known. I calculated (Reference 2) the PCI for ping sequences of varying lengths on the assumption that pings are classified independently one by one with final judgment being based on the character of the entire string. I found that my answers provided only upper bounds to Stradling's values. The conclusion was that the decision on the previous ping is not independent of the decision on the present ping. To account for the lack of agreement it was decided to introduce a degree of dependence in the simplest way by constructing a Markov chain of decisions.

Let us suppose that a succession of pings is reflected from a target which may be either a submarine S or some other object N . The process of reflection imposes the characteristics of the target on the reflected sound pulse so that, initially, we may regard a sequence of pings as containing either the message

$$S, S, S, \ldots S, S$$

if reflected by a submarine, or else the message

$$N, N, N, \ldots N, N$$

if reflected by a non-submarine. The effect of randomness in the environment and imperfections in the recognition system (whether human or automatic) is to transform the above sequences into sequences containing mixtures of symbols such as

$$S, S, N, S, S, S, N, N, S, N, S, S, S, N \quad .$$

Let us denote the sequence called out by the recognition system by

$$Y_1, Y_2, \ldots, Y_n, \ldots$$

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where Y_i ($i = 1, 2, \ldots$) is a random variable that can assume the values S and N. A fixed but unspecified realization of Y_i shall be denoted by y_i. The origin of an observed sequence $y_1, y_2, \ldots, y_n$ will be judged by the maximum likelihood rule: if the probability of the sequence on the hypothesis S is greater than or equal to the probability on the hypothesis N, then the judgment S will be made. The cumulative probability of sequences of length n for which the judgment S is made correctly is called the n-hit probability and the cumulative probability of sequences for which the judgment S is made falsely is called the n-false-alarm probability.

Let $P_S(y_1, y_2, \ldots Y_n)$, denote the probability of the sequence on the hypothesis S. $P_S(Y_1, Y_2, \ldots, Y_n)$, being a function of a sequence of random variables, is then a random variable.

The maximum likelihood rule asserts that the n-hit probability is by

$$P_S\{P_S(Y_1, Y_2, \ldots, Y_n) \geq TP_N(Y_1, Y_2, \ldots, Y_n)\}$$

$$= \Sigma I(y_1, y_2, \ldots, y_n) P_S(y_1, y_2, \ldots, y_n)$$

and that the n-false-alarm probability is given by

$$P_N\{P_S(Y_1, Y_2, \ldots Y_n) \geq TP_N(Y_1, Y_2, \ldots, Y_n)\}$$

$$= \Sigma I(y_1, y_2, \ldots, y_n) P_N(y_1, y_2, \ldots, y_n)$$

where the summation is carried out over all sequences and the indicator function I equals 1 for a sequence $y_1, y_2, \ldots y_n$ for which $P_S(y_1, y_2, \ldots y_n) \geq TP_N(y_1, y_2, \ldots, y_n)$ and 0 otherwise. Here T is a threshold that depends on a priori probabilities and the loss function. The notation $P_S\{\ \}$ is used to denote the probability on the S hypothesis of the event enclosed in the braces.

FORMULAS IN THE MARKOVIAN CASE

On the assumption that $Y_1, Y_2, \ldots, Y_n, \ldots$ constitutes a Markov process, the probabilities entering into the above formulas can be expressed quite simply. Let us introduce the following notation. Let p(S) and p(N) = 1 – p(S) be the probabilities that the decision scheme judges an S to be an S or an N respectively. Let q(S) and q(N) = 1 – q(S) be the corresponding probability for judging an N to be an S or an N. Let W(S, S), W(S, N), W(N, S), and W(N, N) be transition

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probabilities. For example, W(S, N) represents the conditional probability that, if the reflector is an S and a given echo was judged by the decision scheme to have been S, the next echo is judged to be N. The corresponding quantities on the hypothesis that the reflector is an N are V(S, S), V(S, N), V(N, S), and V(N, N). Then with the above notation

$$P_S(y_1, y_2, \ldots, y_n) = p(y_1)W(y_1, y_2)W(y_2, y_3) \ldots W(y_{n-1}, y_n)$$

and

$$P_N(y_1, y_2, \ldots, y_n) = q(y_1)V(y_1, y_2)V(y_2, y_3) \ldots V(y_{n-1}, y_n) \quad .$$

Since $W(y_i, y_{i+1})$ and $V(y_i, y_{i+1})$ can assume only four different values we can rewrite the above expressions in the form

$$P_S(y_1, y_2, \ldots, y_n) = p(y_1)W(S, S)^{K(S, S)}W(S, N)^{K(S, N)}$$
$$W(N, S)^{K(N, S)}W(N, N)^{K(N, N)}$$

$$P_N(y_1, y_2, \ldots, y_n) = q(y_1)V(S, S)^{K(S, S)}V(S, N)^{K(S, N)}$$
$$V(N, S)^{K(N, S)}V(N, N)^{K(N, N)}$$

where K(S, S), K(S, N), K(N, S), and K(N, N) represent the number of consecutive pairs (S, S), (S, N), (N, S), and (N, N) in the sequence.

For example, for the sequence

SSNSNNSNNNSSSNN ,

K(S, S) = 3, K(S, N) = 4, K(N, S) = 3, and K(N, N) = 4 .

If the length of a sequence is n , then

$$K(S, S) + K(S, N) + K(N, S) + K(N, N) = n - 1 \quad .$$

In order to derive formulas for the n-hit probability and the n-false-alarm probability let us partition all sequences of length n into four sets E(S, S), E(S, N), E(N, S), and E(N, N). The set E(S, N), for example, consists of all sequences starting with S and ending with N . The contributions from the various sets to the n-hit probabilities are*

*See Appendix for derivation.

1. $\underline{E(S, S)}\ K_0 = N - 1 - K - 2L$

$$I(S, S, \ldots, S)\, p(S)\, W(S, S)^{(N-1)}$$

$$+ \sum_{L=1}^{\lceil (N-1)/2 \rceil} \sum_{K=0}^{N-2L-1} I(L, K) \binom{K+L}{L} \binom{K_0+L-1}{L-1}$$

$$p(S) W(S, S)^K W(S, N)^L W(N, S)^L W(N, N)^{K_0}$$

2. $\underline{E(S, N)}\ K_0 = N - 2 - K - 2L$

$$\sum_{L=0}^{\lceil (N-2)/2 \rceil} \sum_{K=0}^{N-2L-2} I(L, K) \binom{K+L}{L} \binom{K_0+L}{L}$$

$$p(S) W(S, S)^K W(S, N)^{(L+1)} W(N, S)^L W(N, N)^{K_0}$$

3. $\underline{E(N, S)}\ K_1 = N - 2 - K - 2L$

$$\sum_{L=0}^{\lceil (N-2)/2 \rceil} \sum_{K=0}^{N-2L-2} I(L, K) \binom{K+L}{L} \binom{K_1+L}{L}$$

$$p(N) W(S, S)^{K_1} W(S, N)^L W(N, S)^{(L+1)} W(N, N)^K$$

4. $\underline{E(N, N)}\ K_1 = N - 1 - K - 2L$

$$I(N, N, \ldots, N) p(N) W(N, N)^{(N-1)}$$

$$+ \sum_{L=1}^{\lceil (N-1)/2 \rceil} \sum_{K=0}^{N-2L-1} I(L, K) \binom{K+L}{L} \binom{K_1+L-1}{L-1}$$

$$p(N) W(S, S)^{K_1} W(S, N)^L W(N, S)^L W(N, N)^K \quad .$$

In the above formulas, $I(L, K)$ denotes the value of the indicator function for sequences characterized by parameter values L and K.

A corresponding list could be written down for n-false-alarm probability contributions in the four different cases. There is, however, no need since the formulas are obtainable from the ones above by replacing $p(S)$ and $p(N)$ by $q(S)$ and $q(N)$ and by replacing $W(S, S)$, $W(S, N)$, $W(N, S)$ and $W(N, N)$ by $V(S, S)$, $V(S, N)$, $V(N, S)$, and $V(N, N)$.

CONVERGENCE

Let H_n denote the n-hit probability and F_n the n-false-alarm probability. We shall show that as n becomes large H_n and F_n approach limiting values H and F and that these numbers are either 0 or 1 .

If the Markov chain approaches limiting probabilities μ and $(1-\mu)$ under the S hypothesis and ν and $(1-\nu)$ under the N hypothesis, then for large n , approximately,

$$P_S(y_1, y_2, \ldots, y_n) \approx \mu^k (1-\mu)^{n-k}$$

for a sequence with k S-decisions and n-k N-decisions.

Therefore

$$P_\nu \{P_S(Y_1, \ldots, Y_n) \geq P_N(Y_1, \ldots, Y_n)\} =$$

$$P_\nu \{\log P_S(Y_1, \ldots, Y_n) \geq \log P_N(Y_1, \ldots, Y_n)\} \approx$$

$$P_\nu \{\frac{k}{n} \log \frac{\mu}{\nu} + \frac{n-k}{n} \log \frac{(1-\mu)}{(1-\nu)} \geq 0\} \approx$$

$$P_\nu \{\mu \log \frac{\mu}{\nu} + (1-\mu) \log \frac{(1-\mu)}{(1-\nu)} \geq 0\} = 1 \quad .$$

The last equality arises from the universally true inequality

$$\sum_{i=1}^{n} p_i \log p_i/q_i \geq 0$$

for all pairs of sets of positive numbers $\{p_1, p_2, \ldots, p_n\}$ and $\{q_1, q_2, \ldots, q_n\}$ whose sums equal 1 ; i.e., for which $p_1 + p_2 + \ldots + p_n = q_1 + q_2 + \ldots + q_n = 1$.

In the above derivation, the notions were used that limiting probabilities exist for the Markov process and that if in fact the target was S , then $k/n \approx \mu$ by the law of large numbers for Markov processes. It is of course apparent that if the hypothesis N is in fact true, then $k/n \approx \nu$ and the inequality is reversed with the consequence that the probability equals 0.

NUMERICAL RESULTS

A computer program was written to calculate the probabilities H_n and $1 - F_n$. This program was applied to the particular numerical case discussed by Stradling and Schumacher (Reference 1) in which a priori probabilities of S and N are given as $P(S) = .565$ and $P(N) = .435$. The 1-hit probability was $H_1 = .671$ and the 1-false-alarm rate uses $F_1 = .408$. Threshold values of $T = P(N)/P(S) = .77$ and $T = 1$ were used. The first case that was tried was that of independence in which the transition matrices were:

$$W = \begin{pmatrix} .329 & .671 \\ .329 & .671 \end{pmatrix}$$

$$V = \begin{pmatrix} .408 & .592 \\ .408 & .592 \end{pmatrix} .$$

The results are shown in Figures 1 and 2 and show an interesting phenomenon of discontinuity. Points for odd and even n seem to be smoothly connected except for points of discontinuity at $n = 12$ and $n = 25$, for example, for the threshold value $T = 1$.

The conjecture stated in the introduction (that a possible explanation of the data presented by Stradling and Schumacher might lie in a lack of independence) is borne out by a calculation of the probability of correct identification $P_n = .565\ H_n + .435\ (1 - F_n)$ for transition matrices

$$W = \begin{pmatrix} .5 & .5 \\ .329 & .671 \end{pmatrix}$$

$$V = \begin{pmatrix} .592 & .408 \\ .5 & .5 \end{pmatrix}$$

on the S hypothesis. Figure 3 contains a plot of values of P_n calculated on the Markovian assumption and superposed on the plot of Figure 9 of Reference 1 and shows good agreement. These matrices say about the decision process that if the last symbol was called incorrectly the

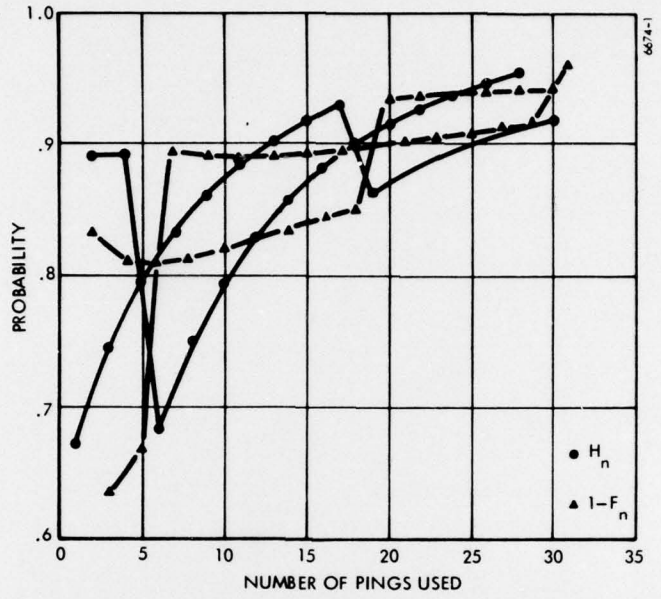

Figure 1. Plot of n-hit probability case of independence with T=0.77.

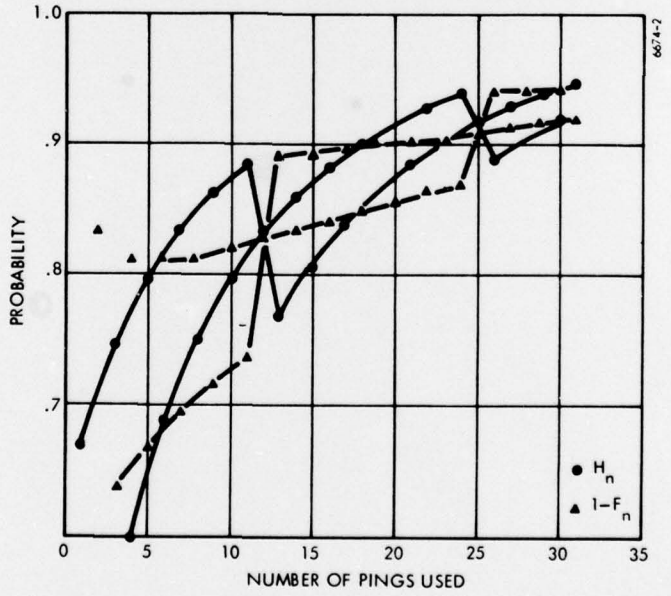

Figure 2. Plot of n-hit probability case of independence with T=1.

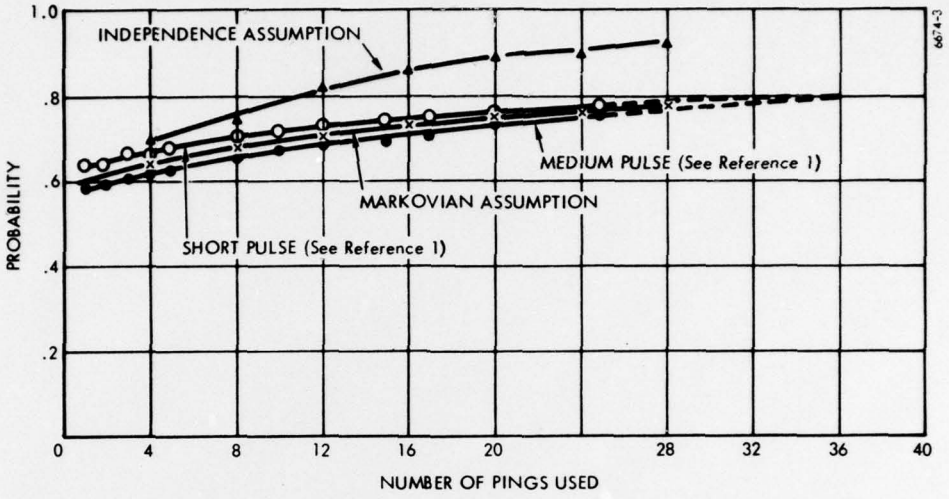

Figure 3. Probability of correct identification versus number of pings used.

present symbol is no more likely to be called correctly than incorrectly. If the last symbol was called correctly, however, the assignment of the present symbol is unaffected, i.e., is made independently of the last symbol.

CONCLUSIONS

The agreement demonstrated in Figure 3 between the observed multi-ping hit and false-alarm rates indicates that the Markovian assumption provides a tenable model for the way that n-ping decisions were actually arrived at. It should be noted that the decision procedure could be improved by enforcing ping-to-ping independence in decisions.

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1. Stradling, C.S., and G.P. Schumacher, A Performance Analysis of the TRESI Classification System Based on the Receiver Operating Characteristic Curves, TM-876, Naval Electronics Laboratory.

2. Kramer, H.P., Classification by Strings of Pings, 66TMP-13, General Electric, TEMPO, Santa Barbara, California, February 1966.

APPENDIX

THE NUMBER OF BINARY SEQUENCES OF TYPE K(N,N), K(N,S), K(S,N), K(S,S)

We divide the calculation into four cases depending on the initial and final symbols in the sequence.

CASE SS

The number of strings of S's = K(S, N) + 1

The number of strings of N's = K(N, S) = K(S, N)

The number of S's = K(S, S) + K(S, N) + 1

The number of N's = K(N, N) + K(N, S)

The number of ways in which the S's can be apportioned among the appropriate number of strings equals the number of compositions

$$\binom{K(S,S) + K(S,N)}{K(S,N)}$$

where the symbol

$$\binom{a}{b} = \frac{a!}{(a-b)!\ b!}$$

is the binomial coefficient.

Similarly, the number of ways in which the N's can be apportioned is

$$\binom{K(N,N) + K(N,S) - 1}{K(N,S) - 1} .$$

Therefore the total number of sequences of the above type is given by the product of the two binomial coefficients

$$\binom{K(S,S,) + K(S,N)}{K(S,N)} \cdot \binom{K(N,N) + K(N,S) - 1}{K(N,S) - 1}$$

CASE SN

The number of strings of S's = K(S, N).

The number of strings of N's = K(N, S).

The number of S's = K(S, S) + K(S, N).

The number of N's = K(N, N) + K(N, S).

Therefore, the number of sequences of the above type is

$$\binom{K(S,S)+K(S,N)-1}{K(S,N)-1} \cdot \binom{K(N,N)+K(N,S)-1}{K(N,S)-1}$$

where now K(S, N) = K(N, S) + 1.

The formulas for the remaining two cases are derived from symmetry considerations.

CASE NS

$$\binom{K(N,N)+K(N,S)-1}{K(N,S)-1} \cdot \binom{K(S,S)+K(S,N)-1}{K(S,N)-1}$$

with K(N, S) = K(S, N) + 1.

CASE NN

$$\binom{K(N,N)+K(N,S)}{K(N,S)} \cdot \binom{K(S,S)+N(S,N)-1}{K(S,N)-1}$$

with K(S, N) = K(N, S).

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TECHNICAL INFORMATION SERIES

AUTHOR	SUBJECT	NO.
H.P. Kramer	Target Classification	66TMP-74
		DATE 18 July 1966
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SUMMARY

The probability of correct target identification is calculated for sequences of pings. The inputs to the calculation are single hit and false alarm probabilities as well as transition probabilities. The inclusion of transition probabilities allows for the possibility that the decision with regard to a given echo may affect the decision about subsequent echoes. Probability of correct target identification for sequences of n pings is calculated by using the maximum likelihood criterion. Numerical results are shown that indicate excellent agreement with the empirical findings of Stradling and Schumacher in connection with the TRESI evaluation. To achieve this agreement it is necessary however to take advantage of the Markovian assumption since in the special case of independence the results of the calculations do not agree with experiment.

KEY WORDS

Markov chains, probability, target classification decision theory, maximum likelihood

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